Content

- A **variable** or an **unknown** is a letter used to represent a number, these can take any values.
- An **expression** is made up numbers and/or letters representing unknown values where there is no equals symbol. For example, $4a + 6$ or $a + b$.
- **Terms** are the parts of expressions separated from each other by + or – signs. They include the + or – sign; if there is no sign, we assume it's +. For example, in $5p + 3q – 4$, there are three terms $+5p$, $+3q$ and -4
- **Coefficients** are the numbers in front of a variable, which multiply the variable. For example, in the term 6m (meaning 6 x m) we say that the coefficient of m is 6; and in $-7n^2$ (meaning $-7 \times n^2$) the coefficient of n^2 is -7. If the coefficient is 1, we don't usually write the 1; so we write w, rather than 1w.
- **Powers** (also called indices or exponents) are superscript numbers after a variable, which show how many times the variable is to be multiplied by itself. For example, t^2 means t x t; or d^5 means d x d x d x d x d. Remember that we don't usually write g^1 but just g.

Take care not to get confused between coefficients and powers!

To **simplify** an expression, you collect together all the terms that are alike (which have the same letter and no others, and are the same power). Remember, each term comes with the sign in front of it. This is often called *gathering like terms.*

Examples
Simplify the following

1) $x + x + x + x + x = 5x$
2) $5e – 2e + e = 4e$
3) $4x + 2y – x + 5y + 6 = 3x + 7y + 6$
4) $3x^2 + 5x + 2x^2 – 4x = 5x^2 + x$
5) $3a + 2a + 7ab + b = 5a + 7ab + b$

When quantities are multiplied together, the result is called a product; we don't write the multiplication sign.

Examples

1) $a \times b = ab$
2) $5 \times 4g = 20g$
3) $3b \times 4c = 12bc$

Linked Prior Topics
Times tables, addition and subtraction

Vocabulary
Variable, unknown, expression, term, coefficient, simplify

Linked Future Topics
Substitution, solving equations and formulae

Content

- An **equation** contains an 'equals' sign and at least one variable. A value can be found for the variable and this is known as solving the equation.
- A **formula** is a rule for working out things by relating variables together in different ways. For example, to work out the area of a rectangle, the formula is A = l x w
  where A = area, l = length and w = width
- A **function** changes one set of numbers into another; for example, this could be a sequence or mapping.
- An **identity** looks like a formula but it is true for all values; it shows different ways of writing the same thing. The symbol is ≡

Examples

1) $a + b \equiv b + a$
2) $(x + 1)^2 \equiv x^2 + 2x + 1$

Substitution is where you replace a letter with a number in a formula or expression, to calculate a value.

Examples:

1) Find the value of 5c when c = 4
5 x 4 = 20

2) Evaluate $3a^2$ when a = 5
3×5^2
3 x 25 = 75

3) The velocity of a car is given by **v = u + at**, find value of v when u=10, a= -2 and t=4
v = u + at
v = 10 + -2 x 4
v = 10 - 8
v = 2

Linked Prior Topics
Bidmas

Vocabulary
Equation, formula, identity and substitution

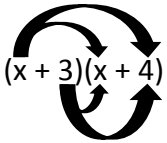
Linked Future Topics
Solving and forming equations, rearranging formula

Expanding

To **expand** double brackets, each term in one bracket needs to be multiplied by each term in the other bracket.

Examples:

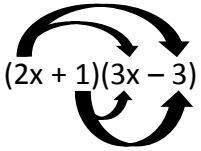
1) Expand and simplify $(x + 3)(x + 4)$



$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

2) Expand and simplify $(2x + 1)(3x – 3)$



$= 6x^2 – 6x + 3x – 3$

$= 6x^2 – 3x – 3$

Some students find it easier to use grid multiplication, like this

	x	+3
x	x^2	+3x
+4	+4x	+12

	2x	+1
3x	$6x^2$	+3x
-3	-6x	-3

An expression is said to be **quadratic** if the highest power of x is 2.

Factorising

Factorising quadratic expressions of the type $ax^2 + bx + c$, where a, b and c are integers, always gives two brackets. If a = 1, we get (x)(x).

For example:

sum product

1) Factorise $x^2 + 5x + 6$

We need two numbers that multiply together to make +6 and add together to make +5.
These numbers must be +3 and +2 and so these are the numbers that we put inside the brackets.
The answer will be (x + 3)(x + 2).

2) Factorise $x^2 – 4x – 21$

We need two numbers that multiply together to make -21 and add together to make -4.
These numbers must be +3 and -7.
The answer will be (x + 3)(x – 7).

3) Factorise $x^2 – 49$

This is a special case where both terms are squares and they are separated by a subtraction sign. It is called the **difference of two squares**. To factorise, find the square root of each term, one bracket will have a minus and one a plus.
The answer will be (x + 7)(x – 7).

Linked Prior Topics

Multiplication, expanding and factorising single brackets, calculating with negatives

Vocabulary

Expanding, factorising, simplify, expression, term, factor, quadratic, sum, product

Linked Future Topics

Solving quadratic equations, cubic equations, graphs of quadratic and cubic equations

Content

To **expand** a **single** bracket, the term on the outside of the bracket needs to be multiplied by each term on the inside of the bracket.

Examples:

1) Expand $2(3m + 5)$
$= \underline{6m + 10}$

2) Expand $4r(2r – 3)$
$= \underline{8r^2 – 12r}$

3) Expand and simplify $2(4m + 3) + 3(5m + 2)$
$= 8m + 6 + 15m + 6$
$= \underline{23m + 12}$

4) Expand and simplify $3(5m + 4) – 2(m + 3)$
$= 15m + 12 – 2m – 6$
$= \underline{13m + 6}$

Some students find it easier to use grid multiplication, like this

	3m	+5
2	6m	+10

	2r	-3
4r	$8r^2$	-12r

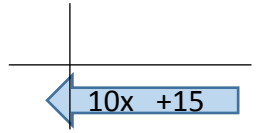
Factorising is the opposite of expanding brackets. To factorise, find common factors to take outside the bracket – these may be numbers, letters or both.

Examples:

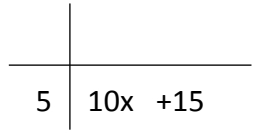
1) Factorise $10x + 15$
- Take the highest common factor outside the brackets, here it would be 5.
- Then complete the brackets by asking what does 5 need to be multiplied by, to get each term in the original expression?
- The factorised expression is $\underline{5(2x + 3).}$

2) Factorise $6x^2 – 9x$
- Take the highest common factor outside the brackets, here it would be 3, and any letters that the terms have in common, here it would be x.
- Then complete the brackets by asking what does 3x need to be multiplied by to get each term in the original expression.
- The factorised expression is $\underline{3x(2x - 3).}$

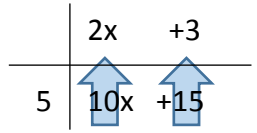
Some students find it easier to use a grid



Common factor is 5



Now fill in the other factors of each term



Answer $10x + 15 \equiv 5(2x + 3)$

Linked Prior Topics
Multiplication, highest common factors

Vocabulary
Expanding, factorising, simplify, expression, term, factor

Linked Future Topics
Expanding double brackets, factorising and solving quadratic equations

Content

A formula usually has a single variable on one side of the equals sign. This is called the 'subject of the formula'. Sometimes you will want to rearrange the formula so that one of the other variables becomes the subject. To do this, you use inverse operations (in a similar way to solving equations) in order to isolate the new subject.

Example 1

Make r the subject of $C = 2\pi r$.

To isolate r, divide by 2π.

$$\frac{C}{2\pi} = r$$

We often write formulae with the subject on the left-hand side, so this becomes

$$r = \frac{C}{2\pi}$$

Example 2

Make x the subject of $y = \frac{x}{5} + 3$.

To isolate x, start by subtracting 3.

$$y - 3 = \frac{x}{5}$$

Next, multiply by 5 (remember to multiply each term of the left-hand side).

$$5(y - 3) = x$$

$$x = 5(y - 3)$$

Example 3

Make r the subject of $V = \frac{1}{3}\pi r^2 h$.

To start, isolate r^2 by multiplying by 3 and then dividing by πh.

$$3V = \pi r^2 h$$

$$\frac{3V}{\pi h} = r^2$$

Now we square root both sides.

$$\sqrt{\frac{3V}{\pi h}} = r$$

$$r = \sqrt{\frac{3V}{\pi h}}$$

Example 4

Make x the subject of $3x + 5 = y - ax$.

When a formula contains the new subject more than once, start by isolating any terms including it on one side of the equals sign.

Here, add ax and subtract 5.

$$3x + ax = y - 5$$

Now we factorise the side with our new subject.

$$x(3 + a) = y - 5$$

Then divide by the bracket to leave x on its own.

$$x = \frac{y - 5}{3 + a}$$

Linked Prior Topics
Equations, inverse operations, factorising

Vocabulary
Formula, rearranging, inverse, subject, isolate

Linked Future Topics
Differential equations, other applications of mathematics