Addition Example: 369 + 27

Write down all of the column headings you will need, and put the digits into the columns according to their place values

	H	T	U
	3	6	9
+		2	7

Start adding in the right-hand column, and move one column to the left each time. Write any 'carried' digits under the line, and don't forget to add them in, like this

	3	6	9
+		2	7
			6
		1	

Think about it as *"9 + 7 = 16 so put the 6 in the right-hand column, and the 1 under the next column to the left"*

Then we move on to the next column

	3	6	9
+		2	7
	3	9	6
		1	

Think about it as *"6 + 2 + 1 that I carried = 9 then 3 + nothing = 3"*

Remember that a question asking for the sum or the total of two or more numbers, it just means ADD THEM UP.

Subtraction Example: 325 – 67

Write down all of the column headings you will need, and put the digits into the columns according to their place values

	H	T	U
	3	4	5
–		6	7

Start subtracting in the right-hand column, and move one column to the left each time. You may need to 'borrow' from the column to the left

	3	$\not{4}^{3}$	$^{1}5$
–		6	7
			8

Think about it as *"5 – 7 won't go, so borrow ten from the next column, to make it 15 – 7"*

Then move to the next column to the left

	$\not{3}^{2}$	$^{1}3$	
		$\not{4}$	$^{1}5$
–		6	7
	2	7	8

Think about it as *"3 – 6 won't go, so borrow ten from the next column, to make it 13 – 6 then 2 – nothing = 2"*

Remember that a question asking for the difference between two numbers, it means SUBTRACT.

Key Words

Place value: thousands / hundreds, tens / units / decimal point / tenths / hundredths *&c*
Addition / plus / the sum / the total
Subtraction / minus / the difference

Linked topics

Place value
Decimals
Directed number

Putting extra zeroes means 'there's no value at this decimal place'. In a number like 2.50, there's 'no hundredths'. So it's ok to write 2.5 as 2.50 or 2.500, etc. But DO NOT take 2.5 and make it 20.5!

Adding Decimals

To add decimals, follow these steps:

1. Write down the numbers, one under the other, with the decimal points lined up.
2. Put in zeros so the numbers have the same length (see below for why that is
3. Then add using column addition, remembering to put the decimal point in the answer.

Example: Add 1.452 to 1.3

Line the decimals up:	"Pad" with zeros:	Add:
1.452 + 1.3	1.452 + 1.300	1.452 + 1.300 2.752

Subtracting Decimals

To subtract decimal numbers:

1. Put the numbers in a vertical column aligning the decimal points.
2. Subtract each column, starting on the right and working left. ...
3. Place the decimal point in the answer directly below the decimal points in the terms.
4. Check the answer by adding.

Example: What is 7.368 – 1.15 ?

Line the decimals up:	"Pad" with zeros:	Subtract:
7.368 – 1.15	7.368 – 1.150	7.368 – 1.150 6.218

Multiplying Decimals

- Multiply normally as you would with integers using column multiplication , ignoring the decimal points.
- Then, put the decimal point in the answer: count up how many numbers are after the decimal point in *both* numbers, then the answer should have that many numbers after *its* decimal point.

2.34 ← 2 decimal places
× 1.6 ← + 1 decimal place
1404
234 0
3.744 ← 3 decimal places

.43
× .06
.258
4 3 2 1

0.275 × 0.54 = 0.14850
275
× 54
1100
13750
14850

Dividing by Decimals

- The trick is to change the number we are dividing by to a whole number first, by shifting the decimal point of both numbers to the

7.5 ÷ 0.25 → 750 ÷ 25

30
25) 750

Now we are **dividing by a whole number**, and can continue as normal using bus-stop method .
It is safe to do this if we remember to shift the decimal point of **both numbers** the same number of places.

Example

6 ÷ 0.3 → 60 ÷ 3 = 20

Example

0.546 ÷ 0.07 → 54.6 ÷ 7 = 9.8
09.8
7) 54.6

Linked Prior Topics
Place value, column addition / subtraction of integers, column multiplication of integers, bus-stop division of integers

Vocabulary
calculate, addition, subtraction, multiplication, division, decimal, integer, place value

Linked Future Topics
Estimating using calculations, bounds

Rounding to…

The nearest 10, 100 or 1000

Remember to kept the place value of each number by inserting zeros where applicable.

Look at the number which **represents the place value**, look to the **right**: if this digit is **5 or more** the number **rounds up by 1**.

If the digit is **4 or less,** the number **stays the same.**

E.g. Round **17 839** to the nearest 10, 100 & 1000

(i) Nearest 10 – 17 840

(ii) Nearest 100 – 17 800

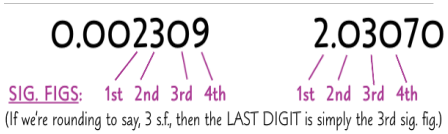
(iii) Nearest 1000 – 18 000

Decimal places (d.p.)

1. **Identify the position** of the decimal place to be rounded to, e.g. 2d.p. would be the 2nd digit after the decimal place. Draw a **cut-off line** after this.
2. Then look to the **right** of this digit, this is called the **decider digit;** this number now decides whether the decimal place is rounded up or kept the same.
3. If the **decider digit** is **5 or more** then **round the digit up**.
4. If the **decider digit** is **4 or less** then **leave the digit as it is**.

Significant figures (s.f.)

1. The **first significant number** is the first digit of a number which isn't zero.
2. The **2nd, 3rd, digits** follow immedi*ately after the 1st , regardless of zeros.*



3. When rounding numbers the **place value** of each digit must be the same.

Estimating

When estimating you are **not guessing;** you are making the numbers 'easier' for you to work out the calculation. To estimate, you need to

1. Round each number to **1s.f**.
2. Then calculate the sum using the **hierarchy of operations.**

Example 1:

Estimate $\frac{29.91 \times 38.3}{3.1 \times 3.9}$

- Round to 1s.f.

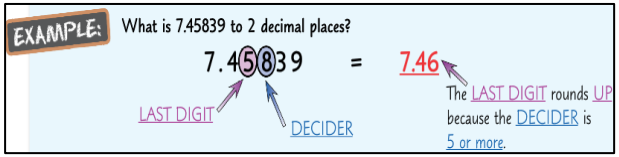
$$\frac{30 \times 40}{3 \times 4} = \frac{1200}{12} = 100$$

Example 2:

Estimate $\frac{23.43 \times 4.3}{0.483}$

- Round to 1s.f.

$$\frac{20 \times 4}{0.5} = \frac{80}{0.5} = 160$$



Special case:

Round 23.999 to 2d.p. = 24.00

As the 3rd 9 rounds up the 2nd 9 which rounds up the 1st 9 due to the 9 turning into a '10' and insert zeros.

		to 3 s.f.	to 2 s.f.	to 1 s.f.
1)	54.7651	54.8	55	50
2)	0.0045902	0.00459	0.0046	0.005
3)	30895.4	30900	31000	30000

Estimating square roots

1) Find the two consecutive square numbers either side.

E.g. $\sqrt{48}$

The two consecutive square numbers are 36 and 49.

2) Find the **square roots** of these two numbers: $\sqrt{36} = 6$ *and* $\sqrt{49} = 7$.

Therefore the estimate of $\sqrt{48}$ would be between 6 & 7.

This can then lead onto estimating the value, as 48 is close to 49, I would estimate that $\sqrt{48}$ = 6.9

Linked Prior Topics:

- Place value

Vocabulary:

- Significant figures
- Decimal places
- Estimate
- Cut-off line
- Decider digit

Linked Future Topics:

- Rounding answers e.g. area of a circle
- Bounds & error intervals

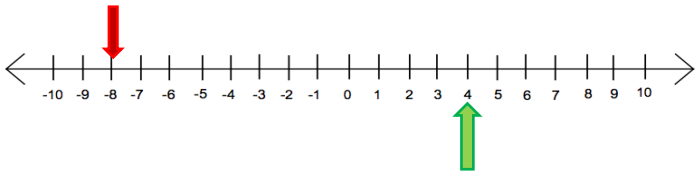
UNIT 1a: DIRECTED NUMBER (POSITIVE & NEGATIVE NUMBERS) | **GCSE HIGHER** | **MATHEMATICS DEPARTMENT**

Negative numbers are numbers that are **less than zero**.

Addition and Subtraction

Example : - 8 + 12

When adding and subtracting with negative numbers, you should use a number line. Start at the first number given in the sum (here, it's -8).



Then, think about whether you are adding or subtracting your number. If you're adding the number needs to get bigger, so you move to the right each time. If you're subtracting, the number must need to be smaller, so you move left. We need to add 12 in our example, so add 8 to get back to zero, then add on in 1's until you get to 12...

The number you end up on is your answer! **- 8 + 12 = 4**

Be aware of circumstances where there are two signs in the middle of the sum:-

Example 1: -5 + - 3

Where the signs in the middle are different, the resulting sum becomes a subtraction so the above example would be -5 – 3 which equals -8.

Example 2: -6 - - 3

Where the signs in the middle are the same, the resulting sum becomes an addition so the above example would be -6 + 3 which equals -3.

Multiplication and Division

Rules for **multiplying** with negative numbers:-

Positive x Positive = Positive
Negative x Negative = Positive
Positive x Negative = Negative
Negative x Positive = Negative

Examples

5 x 4 = 20
-3 x -2 = 6
10 x -7 = -70
-8 x 9 = -72

✓ Where the signs are the same, the product of the numbers is positive!

✓ Where the signs are different, the product of the numbers is negative!

Rules for **dividing** with negative numbers:-

Positive ÷ Positive = Positive
Negative ÷ Negative = Positive
Positive ÷ Negative = Negative
Negative ÷ Positive = Negative

Examples

20 ÷ 4 = 5
-6 ÷ -2 = 3
70 ÷ -7 = -10
-72 ÷ 9 = -8

✓ Where the signs are the same, the quotient of the numbers is positive!

✓ Where the signs are different, the quotient of the numbers is negative!

THINK! TWO NEGATIVES DO NOT ALWAYS MAKE A POSITIVE...

Linked Prior Topics
Basic addition and subtraction
Basic multiplication and division

Vocabulary
Negative, positive, directed numbers

Linked Future Topics
All future topics

Example: 9345 ÷ 15

1) Set up your bus stop so that the number you are dividing by is outside (trying to get in).

$$15\overline{)9\,3\,4\,5}$$

2) Write down the times table of the number you're dividing by.

1 x 15 = 15, 2 x 15 = 30, 3 x 15 = 45, 4 x 15 = 60, 5 x 15 = 75,
6 x 15 = 90, 7 x 15 = 105, 8 x 15 = 120, 9 x 15 = 135

3) At each stage, you're asking yourself, "How many of the number outside can fit IN to the next part of the number in the bus stop?" In our example, 15 doesn't fit into 9 at all, so none of the 9 has been used yet. Because of this, we carry it across to turn the 3 into 93.

$$\begin{array}{r} 0 \\ 15\overline{)9\,3\,4\,5} \end{array}$$

4) Next we ask ourselves, "How many 15's fit into 93?" Looking at the times table, 6 x 15 = 90, so 6 fits in. We had 93 though, so 3 has not yet been used and can be moved over to make the 4 become 34.

$$\begin{array}{r} 0\,6 \\ 15\overline{)9\,3\,4\,5} \end{array}$$

5) Continue like this until you have gone through all of the numbers in the bus stop. The number above the bus stop is your answer.

9345 ÷ 15 = 623

Linked Prior Topics

- Multiplication Tables

Vocabulary

- Dividing
- Bus Stop
- Carry

Linked Future Topics

- Division with negative numbers.
- Division with decimals.
- Algebraic Division

Multiplication:

Example: 372 x 24

1) Restate the question in a column multiplication:-

```
  372
x  24
```

2) Firstly, multiply each digit of 372 by the unit digit in 24 ('4') working from right to left:-

```
  372
x  24
 1488
    2
```

NB. Remember to carry to the next column if the product of two numbers is more than 10 and remember to add this on to the next column.

3) On the next row, insert a **zero** on the far right. Then, multiply each digit of 372 by the tens digit in 24 ('2') working from right to left:-

```
  372
x  24
 1488
 7440
```

NB. Remember to carry to the next column if the product of two numbers is more than 10 and remember to add this on to the next column.

4) Add up columns indicated in red below for the final answer:

```
  372
x  24
 1488
 7440
 8928
    1
```

372 x 24 = 8928

NB. As the number you are multiplying by increases by a power of 10, another row of working out is added and an additional zero is added on the right-hand side.

Vocabulary

- Multiplication
- Increase
- Product
- Column

Linked Prior Topics

- Multiplication Tables

Linked Future Topics

- Factors
- Multiples
- Multiplying Fractions

UNIT 1b: INDICES, ROOTS & ORDER OF OPERATIONS

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Indices are the numbers written above a base number, so for 8^2, 8 is the base and 2 is the index

The **index** shows how many times it has been multiplied by itself, so $3^7 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$

e.g. $3^6 \times 3^4 = 3^{6+4} = 3^{10}$, $a^2 \times a^7 = a^{2+7} = a^9$

2) When DIVIDING, you SUBTRACT THE POWERS.
e.g. $5^4 \div 5^2 = 5^{4-2} = 5^2$, $b^8 \div b^5 = b^{8-5} = b^3$

3) When RAISING one power to another, you MULTIPLY THEM.
e.g. $(3^2)^4 = 3^{2\times4} = 3^8$, $(c^3)^6 = c^{3\times6} = c^{18}$

4) $x^1 = x$, ANYTHING to the POWER 1 is just ITSELF.
e.g. $3^1 = 3$, $d \times d^3 = d^1 \times d^3 = d^{1+3} = d^4$

5) $x^0 = 1$, ANYTHING to the POWER 0 is just 1.
e.g. $5^0 = 1$, $67^0 = 1$, $e^0 = 1$

6) $1^x = 1$, 1 TO ANY POWER is STILL JUST 1.
e.g. $1^{23} = 1$, $1^{89} = 1$, $1^2 = 1$

7) FRACTIONS — Apply the power to both TOP and BOTTOM.
e.g. $\left(1\frac{3}{5}\right)^3 = \left(\frac{8}{5}\right)^3 = \frac{8^3}{5^3} = \frac{512}{125}$, $\left(\frac{u}{v}\right)^5 = \frac{u^5}{v^5}$

8) NEGATIVE Powers — Turn it Upside-Down

People have real difficulty remembering this — whenever you see a negative power you need to immediately think: "Aha, that means turn it the other way up and make the power positive".

e.g. $7^{-2} = \frac{1}{7^2} = \frac{1}{49}$, $a^{-4} = \frac{1}{a^4}$, $\left(\frac{3}{5}\right)^{-2} = \left(\frac{5}{3}\right)^{+2} = \frac{5^2}{3^2} = \frac{25}{9}$

9) FRACTIONAL POWERS

The power $\frac{1}{2}$ means Square Root,
The power $\frac{1}{3}$ means Cube Root,
The power $\frac{1}{4}$ means Fourth Root etc.

e.g. $25^{\frac{1}{2}} = \sqrt{25} = 5$
$64^{\frac{1}{3}} = \sqrt[3]{64} = 4$
$81^{\frac{1}{4}} = \sqrt[4]{81} = 3$
$z^{\frac{1}{5}} = \sqrt[5]{z}$

The one to really watch is when you get a negative fraction like $49^{-\frac{1}{2}}$ — people get mixed up and think that the minus is the square root, and forget to turn it upside down as well.

10) TWO-STAGE FRACTIONAL POWERS

With fractional powers like $64^{\frac{5}{6}}$ always split the fraction into a root and a power, and do them in that order: root first, then power: $(64)^{\frac{1}{6}\times5} = (64^{\frac{1}{6}})^5 = (2)^5 = 32$.

Order of operations

B Brackets ( )
I Indices √ ^{2}
D Division ÷
M Multiplication x
A Addition +
S Subtraction –
(This is sometimes written **BODMAS** with **O** standing for order.)

Reciprocal

The reciprocal of any number is
1 ÷ that number
Examples:

the reciprocal of 5 is $\frac{1}{5}$

the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$

the reciprocal of 0.6 is $\frac{5}{3}$

the reciprocal of $1\frac{2}{5}$ is $\frac{2}{7}$

Warning!
Rules 1 & 2 only work for the same base:
$2^2 \times 2^3 = 2^5$
but NOT for different bases like $2^2 \times 5^3$

EXAMPLE: Simplify $(3a^2b^4c)^3$

Just deal with each bit separately:

$= (3)^3 \times (a^2)^3 \times (b^4)^3 \times (c)^3$
$= 27 \times a^{2\times3} \times b^{4\times3} \times c^3$
$= 27a^6b^{12}c^3$

EXAMPLE: Find the reciprocal of $\sqrt{4+6\times(12-2)}$.

$\sqrt{4+6\times(12-2)} = \sqrt{4+6\times10}$ — It's not obvious what to do with the square root — so use Brackets first...
$= \sqrt{4+60}$ — ... then multiply...
$= \sqrt{64}$ — ... then add.
$= 8$ — Take the square root
The reciprocal of 8 is $\frac{1}{8}$. — Finally, take the reciprocal (the reciprocal of a number is just 1 ÷ the number).

BIDMAS

Linked Prior Topics
- Multiplication
- Fractions
- Adding and subtracting (incl. negative numbers)

Vocabulary
- Index (s) / indices (pl)
- Powers = indices = exponent
- Base number - number with an index

Linked Future Topics
- Surds
- Standard form
- Quadratics

Operations are mathematical instructions, and include brackets, indices (also known as powers or order), division, multiplication, addition and subtraction.

Operations must be performed in a particular order.

The acronym **BIDMAS** prescribes this order as follows:-

Brackets
Indices (sometimes written **O**rder)
Division &
Multiplication
Addition &
Subtraction

If there is both division and multiplication, or addition and subtraction, in a calculation, the order in which you do them is less important:

For example, **10 – 3 + 2 = 9** and **4 x 15 ÷ 5 = 12**

Example: $3 \times 5 + (1 + 3)^2$

- Which operations are involved in the calculation?

We have brackets '(1+3)', an index '$(1+3)^2$', a multiplication and an addition.

- In which order should we do them?

BIDMAS says do the brackets first, followed by the index, the multiplication and finally, the addition.

- Do it – step by step!

BIDMAS
$3 \times 5 + (1 + 3)^2$ 1 + 3 = 4, so the calculation becomes …
$3 \times 5 + 4^2$

BIDMAS
$3 \times 5 + 4^2$ $4^2 = 16$ so the calculation becomes …
$3 \times 5 + 16$

BIDMAS
$3 \times 5 + 16$ 3 x 5 = 15, so the calculation becomes …

BIDMAS
$15 + 16$ 15 + 16 = **31**, so that's our final answer.

$3 \times 5 + (1 + 3)^2 = 31$

Linked Prior Topics
Basic addition and subtraction
Basic multiplication and division

Vocabulary
Brackets, indices, order, powers, division, multiplication, addition, subtraction, operation.

Linked Future Topics
All algebra and number topics.

The MULTIPLES of a number are just its times table.

EXAMPLE: Find the first 8 multiples of 13.

You just need to find the first 8 numbers in the 13 times table:

13 26 39 52 65 78 91 104

The FACTORS of a number are all the numbers that divide into it.

There's a method that guarantees you'll find them all:

1) Start off with 1 × the number itself, then try 2 ×, then 3 × and so on, listing the pairs in rows.
2) Try each one in turn. Cross out the row if it doesn't divide exactly.
3) Eventually, when you get a number repeated, stop.
4) The numbers in the rows you haven't crossed out make up the list of factors.

EXAMPLE: Find all the factors of 24.

Increasing by 1 each time

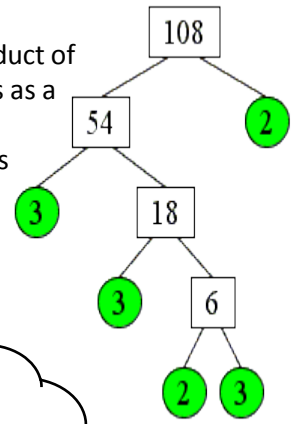
1 × 24
2 × 12
3 × 8
4 × 6
~~5 × –~~
6 × 4

So the factors of 24 are:
1, 2, 3, 4, 6, 8, 12, 24

Factor:
To find a factor of a number you need to find all of the numbers that divide exactly into it.

Multiple:
A multiple of a number is just that number's times table e.g. a multiple of **8** could be **8, 16, 24, 32, 40, 48, 56, 64, 72, 80,** … and so on forever.

Prime Factor Tree:
You could use this to express a number as the product of its factors which are prime numbers. We show this as a prime factor tree.
Don't forget to highlight or circle the prime factors when you find them.
So for this example, we finish by writing
$108 = 2 \times 2 \times 3 \times 3 \times 3 = 2^2 \times 3^3$



A factor of a number will always be smaller than (or equal to) the number itself.

Factors ← 20 → Multiples

1, 2, 4, 5, 10, 20

20, 40, 60, 80, 100, …

A multiple of a number will always be bigger than (or equal to) the number itself.

Linked Prior Topics
- Multiplication
- Division
- Indices

Linked Future Topics
- Factorising
- Simplifying
- Prime factor trees

Vocabulary
- Factor/Factorise
- Multiple
- Common Factors
- Common Multiples
- Highest Common Factor(HCF)
- Lowest Common Multiple (LCM)
- Product
- Prime factors

2 3 5 7 11 13 17 19 23 29 31 37 41 43...

Content

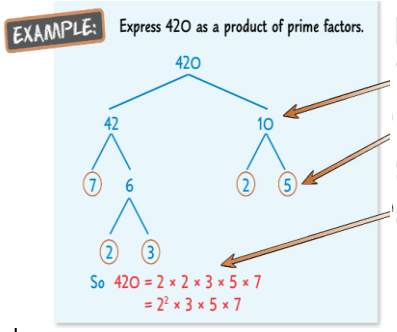
- A prime number (or a prime) is a natural number greater than 1 that has no positive divisors other than 1 and itself.
- Any number which is not itself prime, can be written as the product of prime factors – you can use the factor tree method to do this.
- Any number can be broken down into a string of prime factors all multiplied together – this is called 'prime factor decomposition' or 'prime factorisation'.
- A 'product' is the answer to a multiplication.
- We can write the product of a number in the form of index notation.
  e.g. $2^3 \times 5$

HCF & LCM

When you have found the prime factors of a number you can use this information in order to find out the LCM and HCF.

Common Misconceptions

- 0 and 1 are not considered prime numbers.



To write a number as a product of its prime factors, use the Factor Tree method:

1) Start with the number at the top, and split it into factors as shown.
2) Every time you get a prime, ring it.
3) Keep going until you can't go further (i.e. you're just left with primes), then write the primes out in order.

If there's more than one of the same factor, you can write them as powers.

Finding the HCF

1) List all the PRIME FACTORS that appear in BOTH numbers.
2) MULTIPLY these together to find the HCF.

EXAMPLE: $180 = 2^2 \times 3^2 \times 5$ and $84 = 2^2 \times 3 \times 7$. Use this to find the HCF of 180 and 84.

$180 = 2 \times 2 \times 3 \times 3 \times 5$ $\quad 84 = 2 \times 2 \times 3 \times 7$

2, 2 and 3 are prime factors of both numbers, so

HCF = 2 × 2 × 3 = 12

Finding the LCM

1) List all the PRIME FACTORS that appear in EITHER number.
2) If a factor appears MORE THAN ONCE in one of the numbers, list it THAT MANY TIMES.
3) MULTIPLY these together to give the LCM.

EXAMPLE: $18 = 2 \times 3^2$ and $30 = 2 \times 3 \times 5$. Find the LCM of 18 and 30.

$18 = 2 \times 3 \times 3$ $\quad 30 = 2 \times 3 \times 5$

So the prime factors that appear in either number are: 2, 3, 3, 5 (List 3 twice as it appears twice in 18.)

LCM = 2 × 3 × 3 × 5 = 90

Vocabulary

- Prime number
- Product
- Quotient
- Decomposition
- Positive integer
- Index notation

No matter which numbers you choose at each step, you'll find that the prime factorisation is exactly the same. Each number has a unique set of prime factors.

Linked Future Topics

- Re-occurring decimals
- Coding

Linked Prior Topics

- Factors, Multiples & Prime Numbers
- Multiplication
- Division